

NAG Toolbox for MATLAB

f08ga

1 Purpose

f08ga computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric matrix A in packed storage.

2 Syntax

```
[ap, w, z, info] = f08ga(jobz, uplo, n, ap)
```

3 Description

The symmetric matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo** – string

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

4: **ap(*)** – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work

5.4 Output Parameters

1: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

ap contains the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A .

2: **w**(*) – double array

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

If **info** = 0, the eigenvalues in ascending order.

3: **z**(ldz,*) – double array

The first dimension, **ldz**, of the array **z** must satisfy

if **jobz** = 'V', **ldz** $\geq \max(1, \mathbf{n})$;
ldz ≥ 1 otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'V', then if **info** = 0, **z** contains the orthonormal eigenvectors of the matrix A , with the i th column of Z holding the eigenvector associated with **w**(i).

If **jobz** = 'N', **z** is not referenced.

4: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3 .

The complex analogue of this function is f08gn.

9 Example

```

jobz = 'No vectors';
uplo = 'U';
n = int32(4);
ap = [1;
      2;
      2;
      3;
      3;
      3;
      4;
      4;
      4;
      4];
[apOut, w, z, info] = f08ga(jobz, uplo, n, ap)

apOut =
   -0.3571
    0.1237
   -0.9762
    0.6262
   -1.2472
    7.3333
    0.3660
    0.3660
   -6.9282
    4.0000
w =
   -2.0531
   -0.5146
   -0.2943
   12.8621
z =
    0    0    0    0
info =
    0

```